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Chapter 1
Experimental Validation of the Dual Kalman Filter for Online and Real-Time State and Input Estimation

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Abstract In this study, a novel dual implementation of the Kalman filter is proposed for simultaneous estimation of the states and input of structures via acceleration measurements. In practice, the uncertainties stemming from the absence of information on the input force, model inaccuracy and measurement errors render the state estimation a challenging task and the research to achieve a robust solution is still in progress. Via the use of numerical simulation, it was shown that the proposed method outperforms the existing techniques in terms of robustness and accuracy of displacement and velocity estimations [8]. The efficacy of the proposed method is validated using the data obtained from a shake table experiment on a laboratory test structure. The measured accelerations of the floors of the structure are fed into the filter, and the estimated time histories of the displacement estimates are cross-compared to the true time histories obtained from the displacement sensors.

Keywords Dual Kalman filter • Unknown input • State estimation • Modal identification • TSSID • Experimental validation • Laboratory test

1.1 Introduction

This paper contributes to the procedure of fatigue damage prediction in the entire body of large-scale structures via sparse vibration measurements. An accurate prediction of the fatigue damage demands reliable estimations of the strain time histories in points of interest and their vicinity. Accurate prediction of the strain time histories in turn requires accurate estimates of the states at corresponding degrees-of-freedom of the system [1]. This may be casted as a problem of reliable state estimation under the premise of unknown input measurements. When dealing with a stochastic linear time invariant system, with a known input, state estimates can be calculated in a straightforward fashion. However, in the problem discussed herein the input is unknown. In the last few decades, a number of methods and techniques have been developed to account for the lack of information on the input. Bernal and Ussia present a comprehensive study of sequential deconvolution of inputs from measured outputs [2]. They pose the problem in a deterministic setup, and prove that when there are less inputs than measured outputs the input to the system can be accurately reconstructed. They have shown that the proposed procedure is conditionally stable and the stability criteria is established. However, in practical cases the conditions that must be satisfied to make the deconvolution work are highly likely to be violated.

One of the most recent methods for joint state and input estimation of linear time invariant systems, attempting to incorporate uncertainties, has been developed by Gillijn and De Moore [3]. The method requires a state space model of the system and the second order statistics of the state of the system to recursively furnish the estimates of the input and state. However, when the number of the outputs exceed the order of the model the method suffers from rank deficiency. Lourens et al. [4] have suggested an alteration of the method developed in [3] to alleviate the above-mentioned numerical instabilities. The effectiveness of the proposed adjustment was studied via the joint input force and acceleration estimation of a simulated steel beam, a laboratory test beam and a large-scale steel bridge. It was observed that, even though the method delivers a reasonable estimate of the accelerations, the displacement estimates are affected by spurious low frequency components,

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which were filtered out by using band pass filters. It is noteworthy that in dealing with joint state and parameter estimation, Chatzi and Fuggini [5] have proposed a method to resolve the issues related to the spurious low frequency components in the displacement estimates by including artificial displacement measurements into the observation vector.

Lourens et al. [6] have for the first time applied an augmented Kalman filter (AKF) for unknown force identification in structural systems. It was concluded that the AKF is prone to numerical instabilities due to un-observability issues of the augmented system matrix. Naets et al. proposed an analytical investigation of the stability of the augmented Kalman filter when applied to unknown input and state estimation and demonstrate that the exclusive use of acceleration measurements can lead to unreliable results [7]. In order to alleviate this problem, dummy displacement measurements on a position level are added. The proposed technique is ascertained through numerical investigation and experimental campaign; in both cases it is observed that the AKF based on solely acceleration measurements can lead to unstable results.

To address the shortcomings of the existing methods for state estimation of the structural systems with unknown general inputs, Eftekhar Azam et al. proposed a novel dual Kalman filter (DKF) for state and unknown input estimation via sparse acceleration measurements [8]. It is demonstrated that the successive structure of DKF resolves numerical issues attributed to un-observability and rank deficiency of the AKF. Furthermore, through numerical investigations it is shown that the expert guess on the covariance of the unknown input provides a tool for filtering out the so-called drift effect in the estimated input force and states by GDF. In this paper, an experimental validation of the DKF is pursued; the results of a laboratory experiment are utilized in order to assess the performance of the DKF when real experimental data is used. Moreover, the results obtained by GDF are confronted by those furnished by AKF and GDF.

The paper starts with a section devoted to a brief formulation of the state-space equations for linear time invariant dynamical systems. The next section overviews the DKF, AKF and GDF algorithms and highlights the salient features of each of these approaches in an effort to bring forth their similarities and distinctions. The following section describes the model identification methods used in the current study, and finally the results of the experimental validation of the DKF and its cross-comparison to other filters are recounted.

1.2 Formulation of the Dynamic State Space Equations

The methods and techniques used in this work require existence of an underlying mathematical model of the system which serves as the open-loop estimator of the state of the system. To this end, the linear second order differential equation in continuous time is introduced herein:

$$\mathbf{M\ddot{u}}(t) + \mathbf{C\dot{u}}(t) + \mathbf{Ku}(t) = \mathbf{f}(t) = \mathbf{S_p p}(t) \tag{1.1}$$

where $\mathbf{u}(t) \in \mathbb{R}^n$ denotes the displacement vector and $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M} \in \mathbb{R}^{n\times n}$ stand for the stiffness, damping and mass matrix, respectively. $\mathbf{f}(t) \in \mathbb{R}^n$ is the excitation force, which herein is presented as a superposition of time histories $\mathbf{p}(t) \in \mathbb{R}^m$ that are influencing some degrees-of-freedom of the structure as indicated via the influence matrix $\mathbf{S_p} \in \mathbb{R}^{n\times m}$.

Time discretization should be implemented on the aforementioned equations in order to facilitate their use with the data coming from the sensors. In doing so, first the state space form of the equations are derived, where the state vector encompasses displacement and velocity. The latter allows one to write Eq. (1.1) in the following form and to define the so-called process equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c\mathbf{x}(t) + \mathbf{B}_c\mathbf{p}(t) \tag{1.2}$$

where the system matrices are:

$$\mathbf{A}_c = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$$

$$\mathbf{B}_c = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{S_p} \end{bmatrix}$$

Regarding the measurement equation, the most general case for the observation process is considered where a combination of the displacements, velocities and accelerations are supposed to form the measurement vector:

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$$\mathbf{d}(t) = \begin{bmatrix} \mathbf{S_d} & 0 & 0 \\ 0 & \mathbf{S_v} & 0 \\ 0 & 0 & \mathbf{S_a} \end{bmatrix} \begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \\ \ddot{\mathbf{u}}(t) \end{bmatrix} \tag{1.3}$$

where $\mathbf{S_d}$, $\mathbf{S_v}$ and $\mathbf{S_a}$ denote the selection matrices of appropriate dimension for the displacements, velocities and accelerations, respectively. By using equation of motion, Eq. (1.3) could be transformed so that it forms the observation equation:

$$\mathbf{d}\,(\mathrm{t}) = \mathbf{G}_c\mathbf{x}\,(\mathrm{t}) + \mathbf{J}_c\mathbf{p}\,(\mathrm{t}) \tag{1.4}$$

where the output influence matrix and the direct transmission matrix are:

$$\mathbf{G}_c = \begin{bmatrix} \mathbf{S_d} & 0 \\ 0 & \mathbf{S_v} \\ -\mathbf{S_a}\mathbf{M}^{-1}\mathbf{K} & -\mathbf{S_a}\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$$

$$\mathbf{J}_c = \begin{bmatrix} 0 \\ 0 \\ \mathbf{S_a}\mathbf{M}^{-1}\mathbf{S_p} \end{bmatrix}$$

In this study, the main focus lies in the seismic excitations applied to the base of a structure, hence the excitation term assumes the following form:

$$\mathbf{f}(t) = -\mathbf{M}\ddot{\mathrm{u}}_{\mathbf{g}}\,(\mathrm{t})\,\mathbf{S_p}$$

In Eq. (1.1), it is noteworthy that the coordinate system is relative to the ground displacement, moreover, the subscript **g** refers to the coordinate system, which moves according to seismic ground motions. The matrix $\mathbf{S_p}$ in this case applies the ground accelerations to all floors of the structure.

In practical situations we might use an identified modal model of the structure, hence the equivalent of Eq. (1.1) in modal coordinates is introduced herein. To derive a modal model of the system, Eq. (1.1) is projected to the subspace spanned by the undamped eigenmodes of the system. In this regard, consider the eigenvalue problem corresponding to Eq. (1.1):

$$\mathbf{K\Phi} = \mathbf{M\Phi\Omega}^2 \tag{1.5}$$

Transforming the coordinate system of Eq. (1.1) via the following mapping:

$$\mathbf{u}(t) = \mathbf{\Phi z}(t) \tag{1.6}$$

where $\mathbf{z}(t) \in \mathbb{R}^m$, $\mathbf{\Phi} \in \mathbb{R}^{n\times m}$, and then pre multiplying it by $\mathbf{\Phi}^{\mathrm{T}}$ and dividing the right hand side and left hand side by $\mathbf{\Phi}^{\mathrm{T}}\mathbf{M\Phi}$, considering $\mathbf{\Phi}^{\mathrm{T}}\mathbf{K\Phi}/\mathbf{\Phi}^{\mathrm{T}}\mathbf{M\Phi} = \mathbf{\Omega}^2$ and assuming the damping is proportional, the Eq. (1.6) can be rewritten:

$$\ddot{\mathbf{z}}(t) + \mathbf{\Gamma}\,\dot{\mathbf{z}}(t) + \mathbf{\Omega}^2\mathbf{z}(t) = -\mathbf{P}\,\ddot{\mathrm{u}}_{\mathbf{g}}(\mathrm{t}) \tag{1.7}$$

where the components of jth entry of the diagonal damping matrix $\mathbf{\Gamma}$ are of the form $2\xi_j\omega_j$, in which ξ_j stands for the relevant modal damping ratio. Additionally, $\mathbf{P}$ denotes the modal participation factor $\mathbf{P} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{MS_p}/\mathbf{\Phi}^{\mathrm{T}}\mathbf{M\Phi}$. Note that, a truncated modal space could be substituted in Eq. (1.7)

The recombination of Eqs. (1.3) and (1.4) through use of the relevant matrices, results into the full order state-space equations that are required to implement the input and state estimation algorithm. To derive the modal state-space equations, an eigenvector space must be substituted in Eq. (1.4) hence the following variable transformation would be necessary:

$$\mathbf{x}\,(\mathrm{t}) = \begin{bmatrix} \mathbf{\Phi} & 0 \\ 0 & \mathbf{\Phi} \end{bmatrix} \boldsymbol{\zeta}\,(\mathrm{t})$$

where $\boldsymbol{\zeta}(t)$ is the reduced modal state vector:

$$\boldsymbol{\zeta}\,(\mathrm{t}) = \begin{bmatrix} \mathbf{z}\,(\mathrm{t}) \\ \dot{\mathbf{z}}\,(\mathrm{t}) \end{bmatrix}$$

The reduced modal state-space equation in continuous time will have the following form:

$$\dot{\boldsymbol{\zeta}}\,(\mathrm{t}) = \mathbf{A}_{\mathrm{c}}\boldsymbol{\zeta}\,(\mathrm{t}) + \mathbf{B}_{\mathbf{c}}\ddot{\mathrm{u}}_{\mathbf{g}}\,(\mathrm{t}) \tag{1.8}$$

$$\mathbf{d}\,(\mathrm{t}) = \mathbf{G}_{\mathrm{c}}\boldsymbol{\zeta}\,(\mathrm{t}) + \mathbf{J}_{\mathrm{c}}\ddot{\mathrm{u}}_{\mathbf{g}}\,(\mathrm{t}) \tag{1.9}$$

while the relevant system matrices read:

$$\mathbf{A}_{\mathrm{c}} = \begin{bmatrix} 0 & \mathbf{I} \\ -\boldsymbol{\Omega}^2 & -\boldsymbol{\Gamma} \end{bmatrix},\ \mathbf{B}_{\mathrm{c}} = \begin{bmatrix} 0 \\ -\mathbf{P} \end{bmatrix},\ \mathbf{G}_{\mathrm{c}} = \begin{bmatrix} \mathbf{S}_{\mathbf{d}}\boldsymbol{\Phi} & 0 \\ 0 & \mathbf{S}_{\mathbf{v}}\boldsymbol{\Phi} \\ -\mathbf{S}_{\mathbf{a}}\boldsymbol{\Phi}\boldsymbol{\Omega}^2 & -\mathbf{S}_{\mathbf{a}}\boldsymbol{\Phi}\boldsymbol{\Gamma} \end{bmatrix},\ \mathbf{J}_{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ -\mathbf{S}_{\mathbf{a}}\mathbf{P} \end{bmatrix}$$

To discretize Eqs. (1.8) and (1.9), the sampling rate is denoted by $1/\Delta\mathrm{t}$ and the discrete time instants are defined at $\mathrm{t}_k = \mathrm{k}\,\Delta\mathrm{t}$, for $\mathrm{k} = 1, \ldots, \mathrm{N}$. The discrete state-space equation can be expressed by the following notation:

$$\boldsymbol{\zeta}_{\mathbf{k}+1} = \mathbf{A}\boldsymbol{\zeta}_{\mathbf{k}} + \mathbf{B}\ddot{\mathrm{u}}_{\mathbf{g},\mathbf{k}} \tag{1.10}$$

$$\mathbf{d}_{\mathbf{k}} = \mathbf{G}\boldsymbol{\zeta}_{\mathbf{k}} + \mathbf{J}\ddot{\mathrm{u}}_{\mathbf{g},\mathbf{k}} \tag{1.11}$$

where $\mathbf{A} = \boldsymbol{e}^{\mathrm{A_c}\Delta\mathrm{t}}$, $\mathbf{B} = [\mathbf{A} - \mathbf{I}]\,\mathbf{A}_{\mathbf{c}}^{-1}\mathbf{B}_{\mathrm{c}}$, $\mathbf{G} = \mathbf{G}_{\mathrm{c}}$ and $\mathbf{J} = \mathbf{J}_{\mathrm{c}}$.

1.3 Dual Kalman Filter for Joint Input and State Estimation

In this article, three methods and techniques for the purpose of input and state estimation are cross-compared. First, the main features of each are outlined so that their similarities and distinctions can be summarized. The first algorithm assessed herein is the filter developed by Gillijn and De Moor (GDF) for input and state estimation of linear time invariant systems [3]. The method belongs to the family of recursive Bayesian filters, as such, Bayes' theorem is incorporated into the filter to render the extraction of information on the states from the latest observations possible. To initialize the procedure, the GDF requires an expert guess on the expected value and the covariance of the state at the beginning, then, it recursively estimates the input and state at discrete time instants. First, an estimate of the mean and covariance of the input force is obtained by updating the guess on the input via the GDF input gain, which is similar, but not identical to the Kalman gain. Once the input is estimated, the state vector is subsequently calculated by applying the GDF state gain to the observation novelty. Then, a time update stage follows the procedure which boils down to a mere transition of the state and input through the state-space equations.

Another existing method to achieve recursive state and input estimation is the augmented Kalman filter (AKF). The notion of the augmented state is extensively used in automatic control in order to concurrently estimate the parameters and state of the system. To mitigate some issues attributed to the GDF, this paper also considers the application of the AKF. The AKF turns out liable to numerical instabilities when pure acceleration measurements are fed into it.

Within the frame of state and parameter estimation, an alternative to the augmented formulation is the dual formulation [9]. In dealing with state and parameter estimation of the laminated composites it was shown that the dual formulation outperforms the augmented formulation at the cost of a more complicated implementation [10]. Dealing with the dual estimation and reduced order modelling of linear time varying systems, the dual estimation concept is used to update the reduced subspace constructed by proper orthogonal decomposition [11]. However, in dealing with joint input and state estimation of linear time invariant systems neither the augmented nor the dual formulation lead to nonlinear state space models. The latter motivates the application of the dual formulation of the input-state estimation with coupled use of the

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Table 1.1 The general scheme of the DKF algorithm for input and state estimation

Initialization at time t_0:
$\hat{\mathbf{p}}_0 = \mathbb{E}[\mathbf{p}_0]$ $\mathbf{P}_0^{\mathbf{p}} = \mathbb{E}\left[(\mathbf{p}_0 - \hat{\mathbf{p}}_0)(\mathbf{p}_0 - \hat{\mathbf{p}}_0)^{\mathrm{T}}\right]$
$\hat{\mathbf{x}}_0 = \mathbb{E}[\mathbf{x}_0]$ $\mathbf{P}_0 = \mathbb{E}\left[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^{\mathrm{T}}\right]$
At time t_k, for $k = 1, \ldots, N_t$:
Prediction stage for the input:
1. Evolution of the input and prediction of covariance input: $\mathbf{p}_k^- = \mathbf{p}_{k-1}$ $\mathbf{P}_k^{p-} = \mathbf{P}_{k-1}^{p} + \mathbf{Q}^{\mathbf{p}}$
Update stage for the input:
2. Calculation of Kalman gain for input: $\mathbf{G}_k^{\mathrm{p}} = \mathbf{P}_k^{\mathrm{p}-}\mathbf{J}^{\mathbf{T}}\left(\mathbf{J}\mathbf{P}_k^{\mathrm{p}-}\mathbf{J}^{\mathbf{T}} + \mathbf{R}\right)^{-1}$
3. Improve predictions of input using latest observation: $\hat{\mathbf{p}}_k = \mathbf{p}_k^- + \mathbf{G}_k^{\mathrm{p}}\left(\mathbf{d}_k - \mathbf{G}\,\hat{\mathbf{x}}_{k-1} - \mathbf{J}\mathbf{p}_k^-\right)$ $\mathbf{P}_k^{\mathrm{p}} = \mathbf{P}_k^{\mathrm{p}-} - \mathbf{G}_k^{\mathrm{p}}\mathbf{J}\mathbf{P}_k^{\mathrm{p}-}$
Prediction stage for the state:
4. Evolution of state and prediction of covariance of state: $\mathbf{x}_k^- = \mathbf{A}\,\hat{\mathbf{x}}_{k-1} + \mathbf{B}\hat{\mathbf{p}}_k$ $\mathbf{P}_k^- = \mathbf{A}\mathbf{P}_{k-1}\mathbf{A}^{\mathbf{T}} + \mathbf{Q}^{\mathbf{x}}$
Update stage for the state:
5. Calculation of Kalman gain for state: $\mathbf{G}_k^{\mathbf{x}} = \mathbf{P}_k^-\mathbf{G}^{\mathbf{T}}\left(\mathbf{G}\mathbf{P}_k^-\mathbf{G}^{\mathbf{T}} + \mathbf{R}\right)^{-1}$
6. Improve predictions of state using latest observation: $\hat{\mathbf{x}}_k = \mathbf{x}_k^- + \mathbf{G}_k^{\mathbf{x}}\left(\mathbf{d}_k - \mathbf{G}\mathbf{x}_k^- - \mathbf{J}\hat{\mathbf{p}}_k\right)$ $\mathbf{P}_k = \mathbf{P}_k^- - \mathbf{G}_k^{\mathbf{x}}\mathbf{G}\mathbf{P}_k^-$

Kalman filter. Within the dual estimation scheme, similar to the augmented formulation, the concept of the fictitious transition equation for unknown input is used. Hence, similar to the AKF, in dealing with the DKF the covariance of the fictitious noise must be additionally adjusted to obtain accurate estimates of the state. However, unlike the AKF, the DKF estimates the input and state in two different stages. After initialization, at a first stage the input and the covariance of the input are estimated by applying the Kalman gain to the innovation. At a second stage, the estimated input is fed into a standard Kalman filter which uses the dynamic state-space model to furnish the estimate of the state (Table 1.1).

1.4 Experimental Validation

To assess the performance of the DKF, AKF and GDF in the joint state-input estimation task, the data available from a laboratory structure, shown in Fig. 1.1 has been used. The test is carried out on a three dimensional structure that is comprised of four floors which are stacked onto each other via single span frames. The frame is laterally braced along its strong axis to prevent rotational movements. The structure is subjected to seismic excitations by means of the hydraulic uniaxial shacking table, of the Carleton lab facility in Columbia University, and the response of the structure in terms of the displacements and accelerations is measured on all floor levels by means of laser and MEMS accelerometer sensors, respectively. The methods and techniques that are described and introduced in the previous section require a physical description of the structure, represented in state-space form. In order to obtain the aforementioned numerical model, in this study two approaches are followed; first a modal model of the structure; the second model is established via a recently developed transformation strategy applied on a numerical model obtained by a subspace identification method [12]. Next, a general scheme of each model identification procedure is outlined. The motivation for utilizing both the TSSID and modal model of the structure lies is the numerical divergence of GDF estimates for some sparse sensor setups when the TSSID is used. It is noteworthy that the TSSID model does not make any restrictive assumption on the system matrices; however, the modal model presumes a proportional damping. It is shown that inherent differences between two models can lead to different performances when the state and input estimation is dealt with.

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Fig. 1.1 The four storey shear frame setup on the Carleton laboratory uniaxial shake table

In this paper, the modal characteristics of the structure are estimated using vibration data induced by a measured seismic force applied to its base. The adopted method is based on a least squares minimization of the measure of fit between the Frequency Response Function (FRF) matrix estimated from the measured output acceleration time histories and the FRF matrix predicted by a modal model. In a procedure that can be divided in three main steps, the number of contributing modes and the values of the modal properties are estimated through the FRF estimated from the measured input-output response time histories. In a first step, conventional least squares complex frequency algorithms using the right matrix fraction polynomial model [13, 14] are adopted to acquire estimates of the modal frequencies modal damping ratios and participation factors, and stabilization diagrams furnish a tool to distinguish between the physical and the mathematical modes. In a second step, given that the aforementioned error function is quadratic with respect to the complex mode shapes and the so-called upper and lower residual terms [15], the mode shapes and damping ratios are obtained. These values in most cases are very close to the optimal values. In a third step, the values obtained in the first and second stage are used to efficiently perform the minimization of the objective function. Hence, the computational cost is significantly reduced, when considering that the abovementioned objective function is quadratic with respect to the complex mode shapes.

As a second alternative, a transformed subspace identification technique is used to construct a state-space model for its use with the recursive Bayesian filters. Specifically, the underlying SSI technique is the n4sid algorithm which fits in the class of subspace identification methods [12]. This constitutes an input-output identification method, consisting of QR decomposition of the past and future block Hankel matrices of input and output in order to calculate the main projection matrices. Subsequently, a singular value decomposition (SVD) is executed for extracting the rank of the system and to cast the problem in a standard least-square form. The solution of the latter least squares yields the numerical state space model that fits the input-output data. The disadvantage of Stochastic Subspace Identification methods lies in that the derived state space models are not in their canonical form, i.e. they are not formulated with a minimal parameterization. More importantly, the identified models are not represented in physical coordinates. However, in this study it is necessary to be able to estimate the physical states of the system in the form of displacement, since these are to be subsequently used for fatigue estimation. In this regard, a transformation procedure for achieving such a transformation for structural systems developed by Chatzis et al. [16], hereinafter referred to as the TSSID.

In what follows, the results of the state and input identification by using the DKF, AKF and GDF are presented. First the identified modal properties of the structure are introduced to the filters for deriving the model of the structure.

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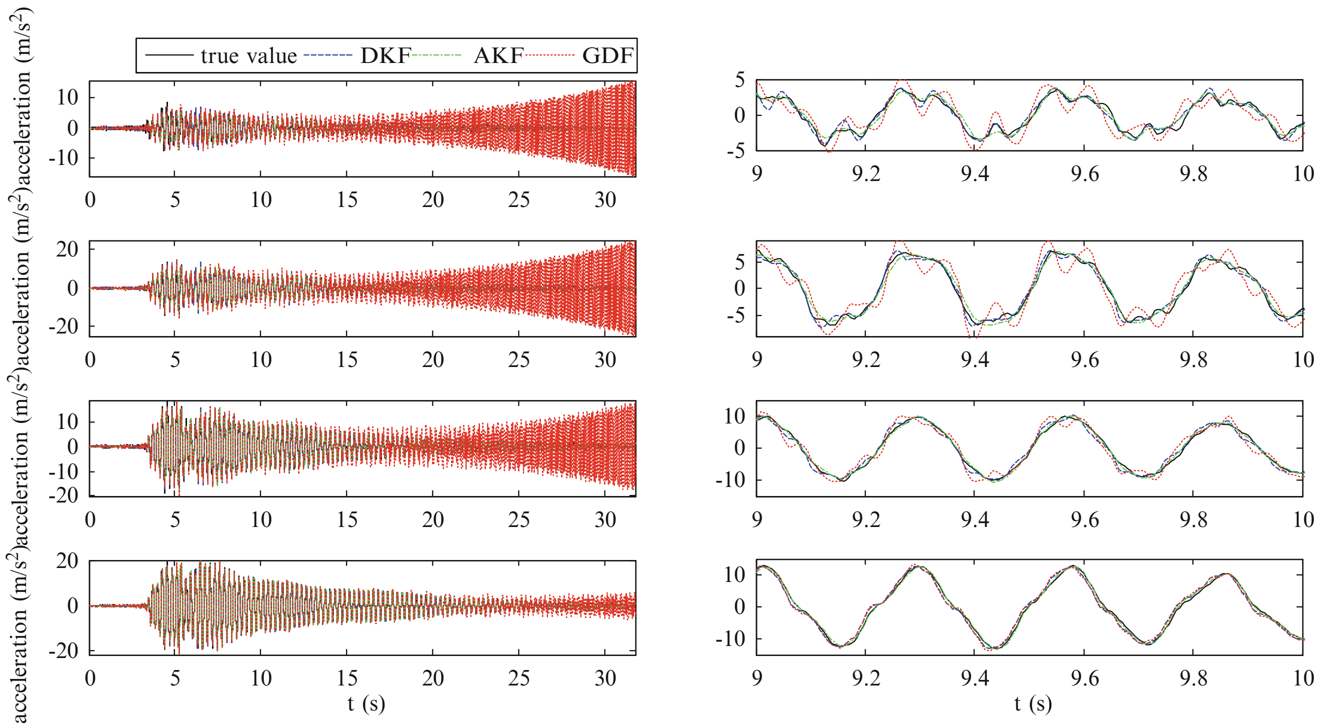

Fig. 1.2 Acceleration time histories estimated by DKF, AKF and GDF of the floors 1, 2, .., 4 from top to bottom, respectively

For the joint state and input estimation for systems with unknown input, the GDF does not require any a-priori assumption on the statistics of the input to the system; on the contrary, the AKF and DKF need the initial value of the mean and covariance of input force to deliver the estimates of the input and states of the system. The value of the covariance of the input force, which plays the role of the regularization parameter, strongly influences the quality of the estimates furnished by the Bayesian filters. In this work, we make use of the L-curve approach in order to adjust the values of the process noise for the state and input estimation [6]. In this way, it turns out that $\mathbf{Q}^{\mathbf{p}} = 10^{-6} \times \mathbf{I}$ and $\mathbf{Q}^{\mathbf{p}} = 1 \times \mathbf{I}$ should be selected for the AKF and DKF respectively when only the measured acceleration time history of the last floor is regarded as observation process. Henceforward $\mathbf{I}$ is an identity matrix of appropriate dimension. The diagonal values of the covariance matrix of the observation noise for acceleration measurement are set to $10^{-3}\ m/s^2$. The process noise $\mathbf{Q}^{\zeta}$ and initial covariance of the state of $\mathbf{P}_0$ are both set to $10^{-20} \times \mathbf{I}$. Figures 1.2 and 1.3 show the acceleration and displacement estimation results provided by the DKF, AKF and GDF for the case where only the acceleration time history of the last DOF is observed. In Fig. 1.2, it is evident that the DKF and AKF provide a reasonable estimate of the observed acceleration time histories; however, the GDF results are affected by numerical instabilities. By increasing the number of the observations the numerical instability can be mitigated, however the emphasis of the current study is on cross-comparison of the three methods when sparse measurements are dealt with. Figure 1.3 shows the results of the displacement estimates provided by the DKF, AKF and GDF. The displacement time history estimated by the GDF is affected by the low frequency components and the same instability trend as in the acceleration estimation. It is observed that the DKF and AKF both provide reasonable estimates of the displacement time history of the test structure.

In what preceded, it was seen that the results of displacement time history estimate furnished by GDF have been affected by low frequency components. In order to get insights on the performance of the GDF filter apart from the drift in the estimates, the modal displacements estimated by GDF have been filtered by using a band pass filter. Figure 1.4 presents the results of such analysis, and confronts the obtained result by DKF and AKF estimates. It is observed that, after implementing the filtering procedure in the estimation results the GDF estimates become more accurate; and match the results furnished by DKF in terms of accuracy and agreement to the target values, nonetheless, the numerical instability caused by low density of the observation sensors still persists.

Next, a model obtained by stochastic subspace identification procedures [5] is incorporated into the filters to investigate the performance of the different schemes when a more realistic model is incorporated into the filters. In this regard, the model obtained through TSSID method, which is described in previous section is introduced into the DKF, AKF and GDF.

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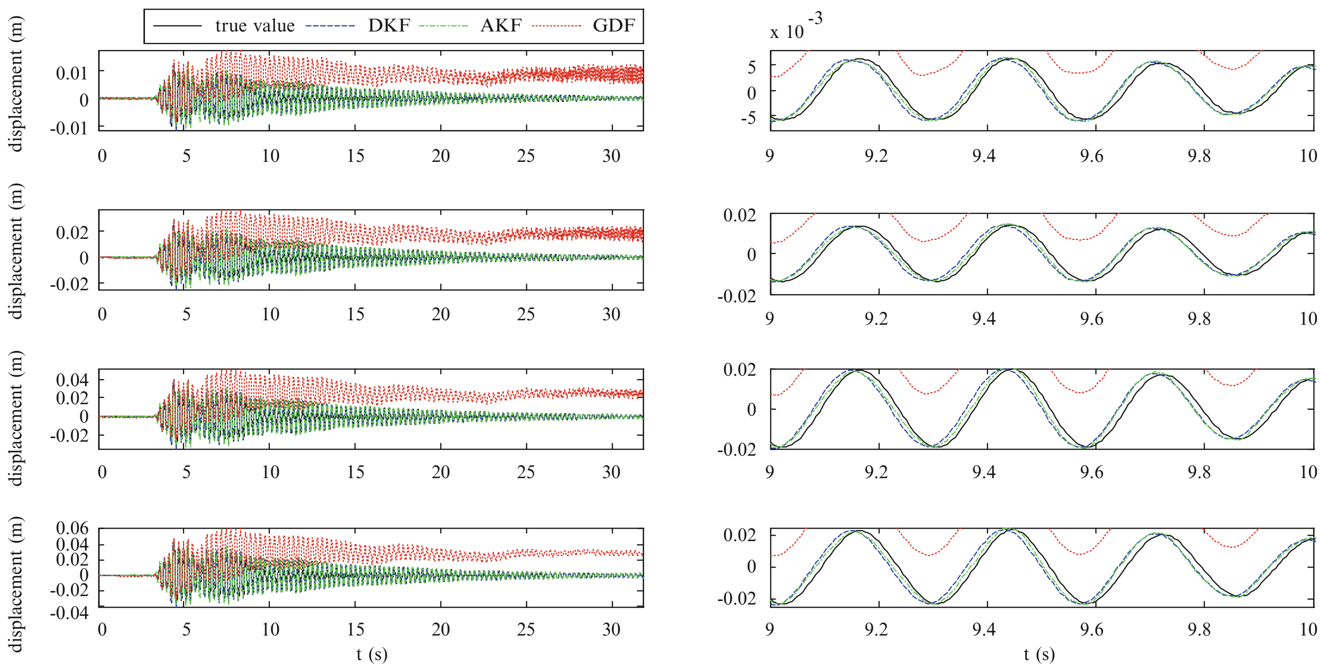

Fig. 1.3 Displacement time histories estimated by DKF, AKF and GDF of the floors 1, 2, .., 4 from top to bottom, respectively

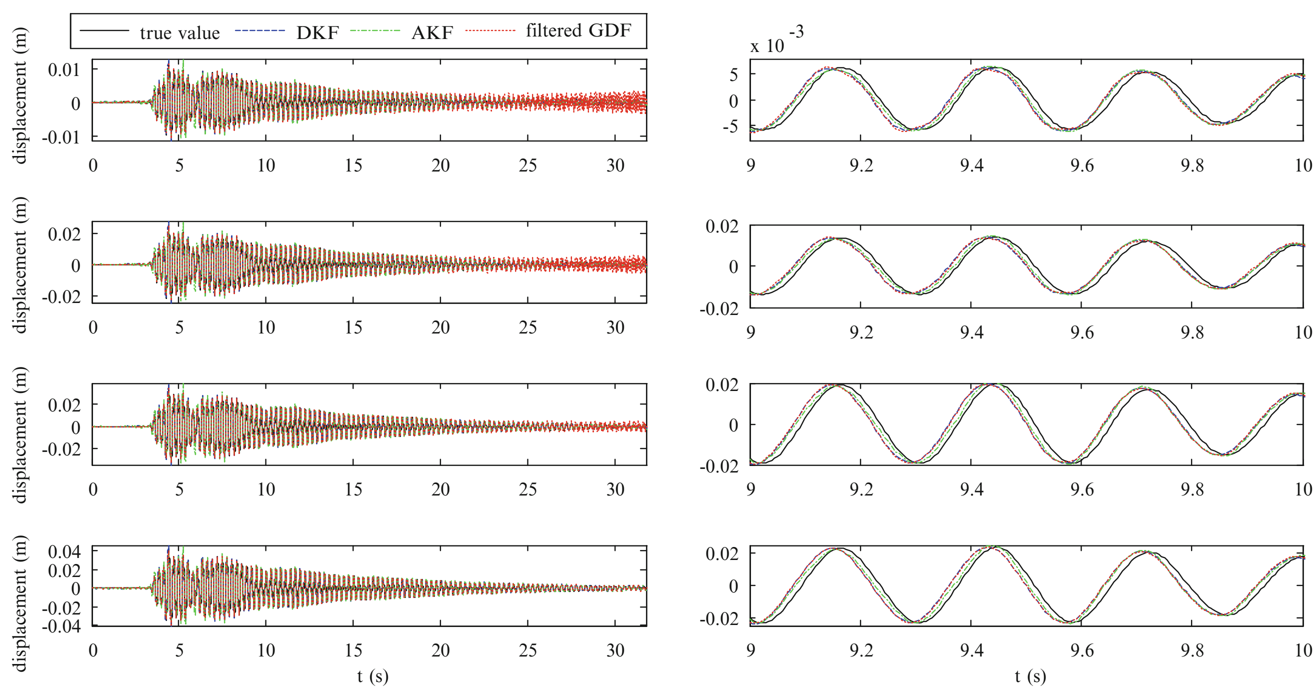

Fig. 1.4 Displacement time histories estimated by DKF, AKF and band pass-filtered GDF for the floors 1, 2, .., 4 from top to bottom, respectively

It was observed that, the GDF fails to provide a solution to the problem when sparse measurements are made. Once again, by increasing the number of the observation DOFs the GDF results can be improved.

Next the capability of the AKF and DKF when sparse acceleration measurements are available is investigated. It is assumed that only the acceleration time history of the last floor is observed, thereafter the performances of the two filter when using modal model and TSSID model are cross-compared. In Figures 1.5 and 1.6 the results of acceleration and displacement estimation furnished by DKF are shown. In Fig. 1.5 it is observed that the acceleration estimations in non-measured DOFs is less accurate than the observed one, additionally, moving from last floor to first floor as the noise to signal

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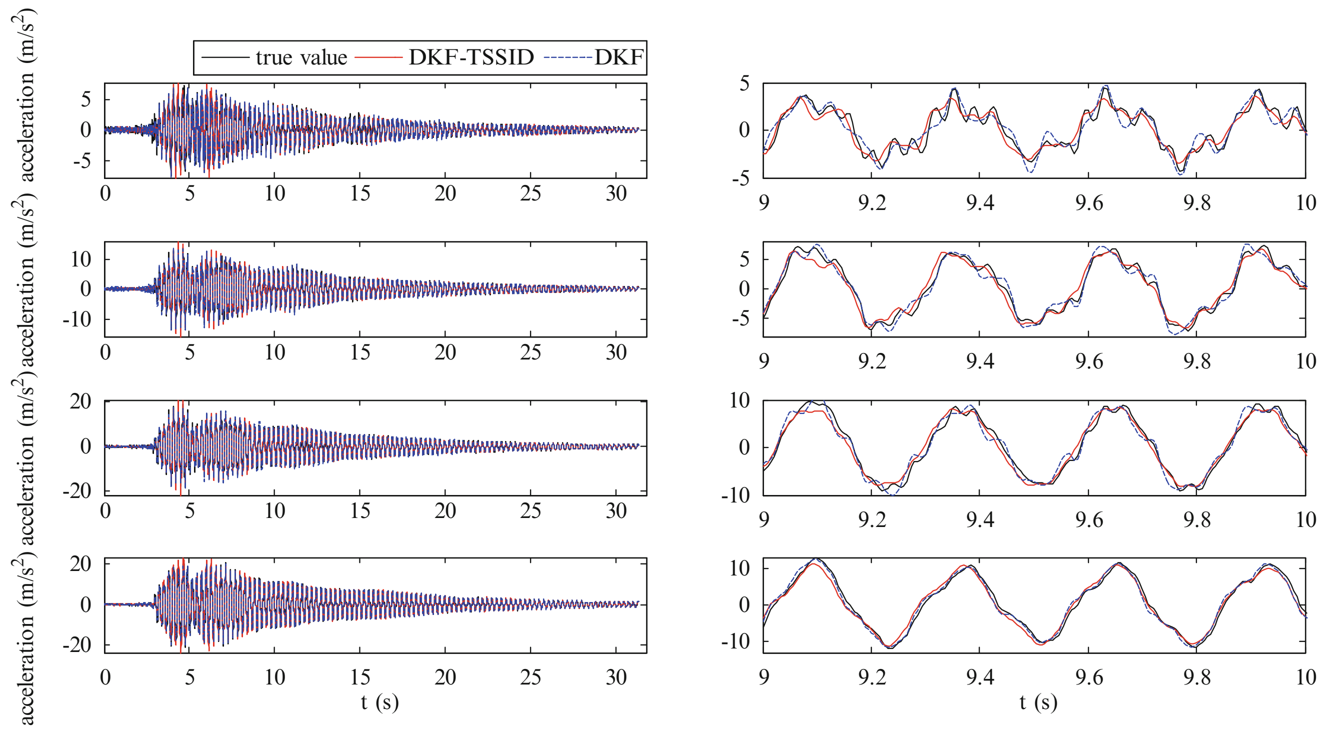

Fig. 1.5 Acceleration time histories estimated by DKF when modal model and TSSID model are used for the floors 1, 2, .., 4 from top to bottom, respectively

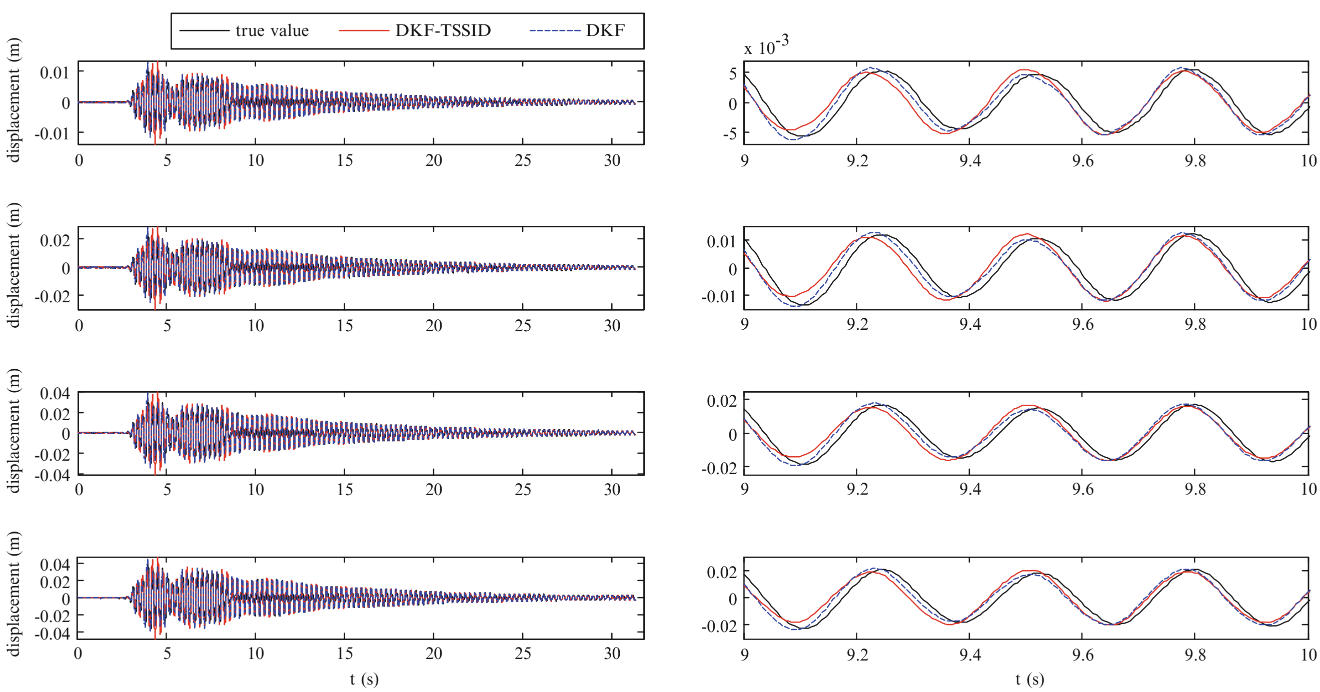

Fig. 1.6 Displacement time histories estimated by DKF when modal model and TSSID model are used for the floors 1, 2, .., 4 from top to bottom, respectively

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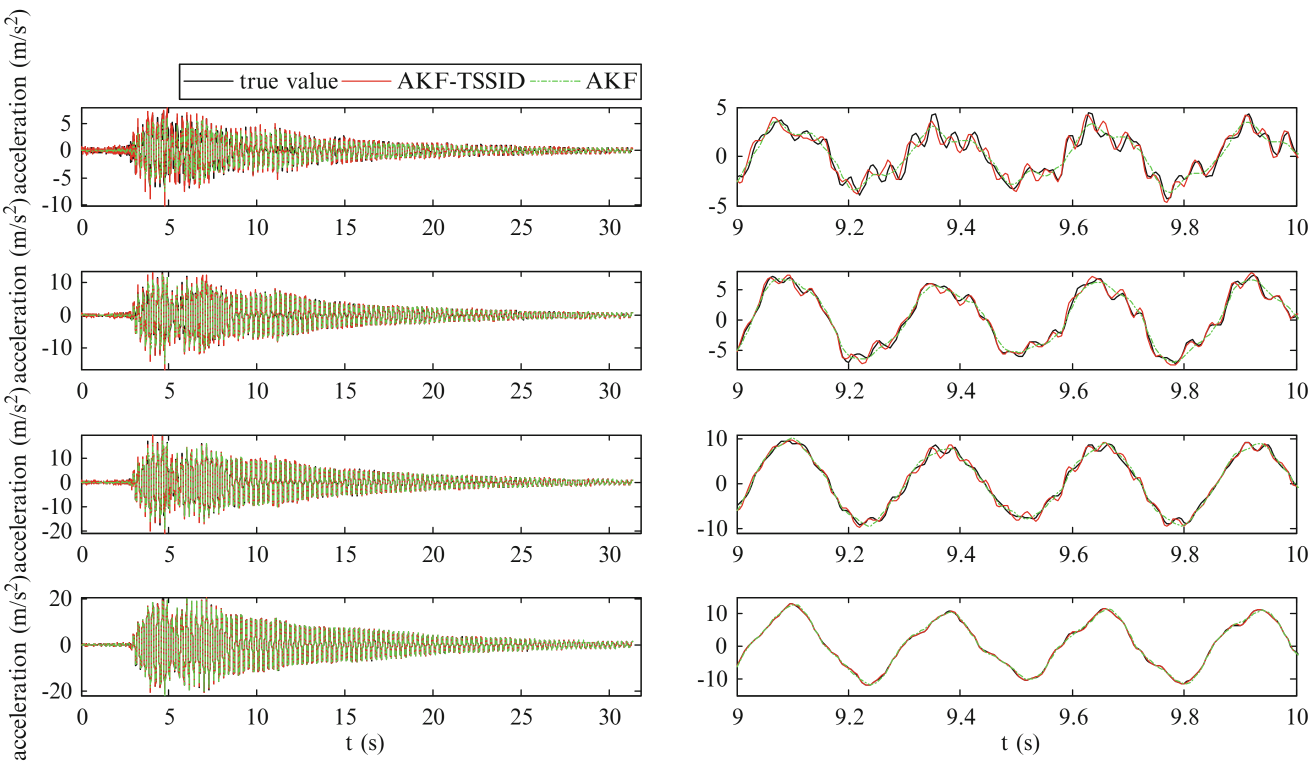

Fig. 1.7 Acceleration time histories estimated by AKF when modal model and TSSID model are used for the floors 1, 2, .., 4 from top to bottom, respectively

ration increases the accuracy of the estimations decrease. Concerning the displacement time histories, it is observed that use of both models lead to the same level of accuracy of displacement estimates.

Dealing with the performance of the AKF when the TSSID model is used Figs. 1.7 and 1.8 present the relevant estimates when the sensor configuration used in the last example is considered. It is observed that use of the TSSID model leads to a more accurate acceleration estimation for un-observed DOFs. Similar to the acceleration estimates obtained by the DKF, the lower the floor the less the accuracy becomes due to the higher signal to noise ratio. Regarding the displacement time histories, both models deliver a similar accuracy.

Finally, the input estimation capability of the three methods is discussed. In Figures 1.9 and 1.10 show the time histories of input and displacement, when DKF, AKD and a band-pass filtered GDF are used. In the latter case, the accelerations of the floor #2 and #4 are assumed as the observation process, due to divergence of the GDF results in the case of a single observation. Moreover, the low frequency drift in the displacement estimates is removed, in a post-processing phase, i.e., not in an online manner, by applying a band pass filter on the modal coordinates. In doing so, first the displacement time histories are projected onto the modal basis, thereafter a band pass filter is used to remove spurious frequencies, which for its lower and upper bounds features the $0.1\,\omega_j,\ j = 1, 2, \ldots, 4$ and Nyquist frequency, respectively. Concerning the band pass filter applied to the input, the lower bound is set $0.1\,\omega_1$. In Figure 1.9, it is observed that the filtered GDF provides a close match between estimated and true input; however, the DKF and AKF fail to provide a reasonable agreement between estimated values and their target ones. However, it is noteworthy that the amplitude of the AKF input estimates is closer to the target value than the DKF. In Figure 1.10 the displacement time histories associated with the inputs are shown. It is observed that the AKF does not provide accurate estimates when tuned so that the amplitude of the identified input is close to target values. Concerning the GDF, it is observed that once the spurious low frequency components are removed by the aforementioned band pass filter (off-line), accurate estimates of the displacement time histories are indeed obtained. When dealing with the DKF, it is observed that the accuracy of the displacement estimates is slightly lower than that of the filtered GDF, however these are delivered on an online manner.. It is important to note that the DKF as well could be tuned, via calibration of the fictitious process noise, to better approximate the unknown input, at the cost of reduced accuracy for the state estimates. However in the fatigues estimation problem discussed herein, the main objective is the estimate of the displacement states in particular and not of the unknown input itself. Therefore, the DKF is the preferred solution herein, given that is also operates in an online manner.

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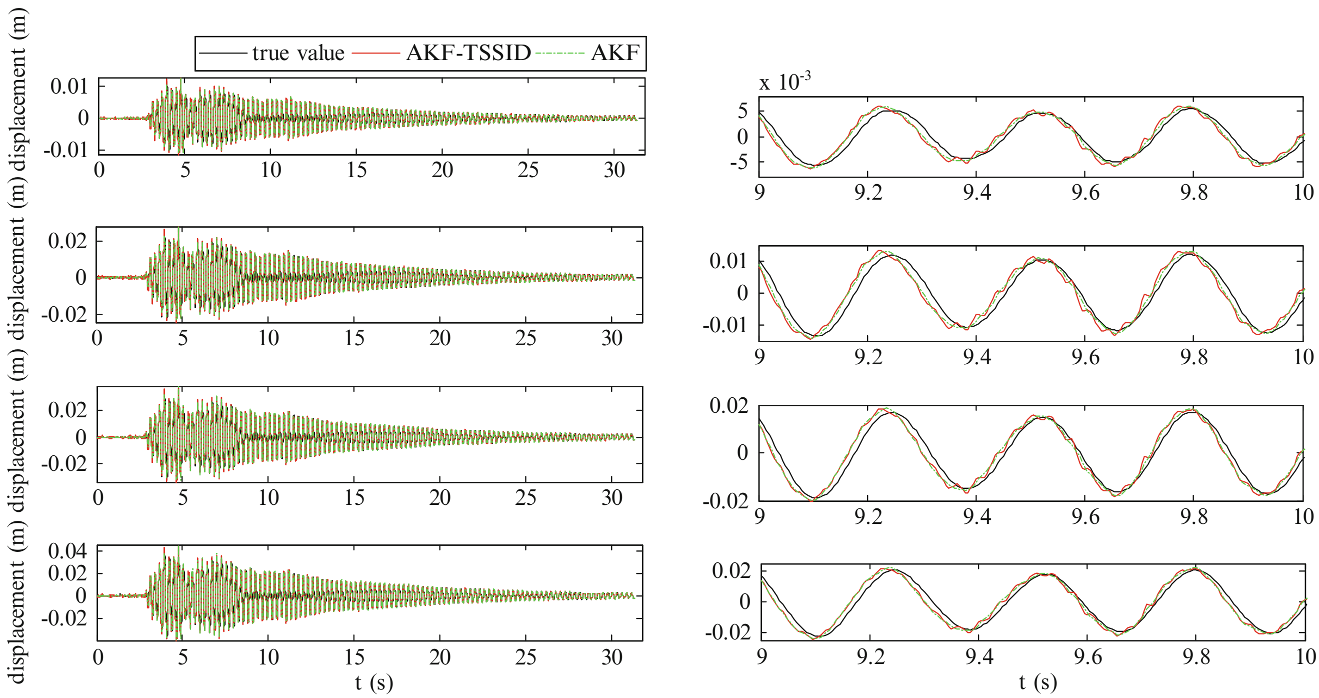
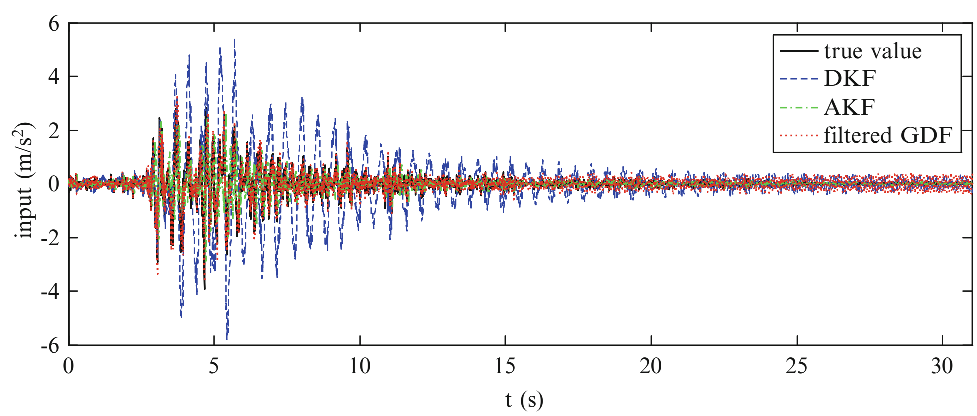


Fig. 1.8 Displacement time histories estimated by AKF when modal model and TSSID model are used for the floors 1, 2, .., 4 from top to bottom, respectively

Fig. 1.9 Input time histories estimated by DKF, AKF and band pass filtered GDF when TSSID model is used for floors 1, 2, .., 4 from top to bottom, respectively. The DKF and AKF are tuned for input estimation rather than the displacement, i.e. $\mathbf{Q}^{\mathbf{p}} = 10^{-3} \times \mathbf{I}$



1.5 Conclusion

In this work the AKF, GDF and a novel dual Kalman filter are applied for calculating the input and displacement time histories of a laboratory test structure tested on a uniaxial shake-table. Concerning the GDF, it is known that the accumulation of the errors arising from integration of noisy accelerations leads to low frequency components in the displacement and input force estimation which must be filtered out. Consequently, in this work the accuracy of the aforementioned estimates after low-pass filtering have been evaluated, which for online purposes is not an optimal consideration. Next, the augmented Kalman filter (AKF), which has been proposed by Lourens et al. [6] for solution of the input and state estimation is considered. The AKF requires introducing the observation process vector into the state transition matrix; it has been shown through theoretical studies that in case of pure accelerations measurements this method suffers from un-observability issues [7]. To address abovementioned issues, a dual formulation of the Kalman filter is considered herein. Numerical investigations have shown that the DKF not only solves the issues stemming from un-observability of the system matrices in the AKF, but also mitigates issues arising from the accumulation of the measurement errors including the drift in displacement and input force estimates [8]. To evaluate the performance of the proposed method a laboratory test structure has served as the examined test case. The measured accelerations of the floors of the structure are fed into the filter, and the estimated time histories of the displacement are confronted by the true time histories obtained from the displacement sensors. In

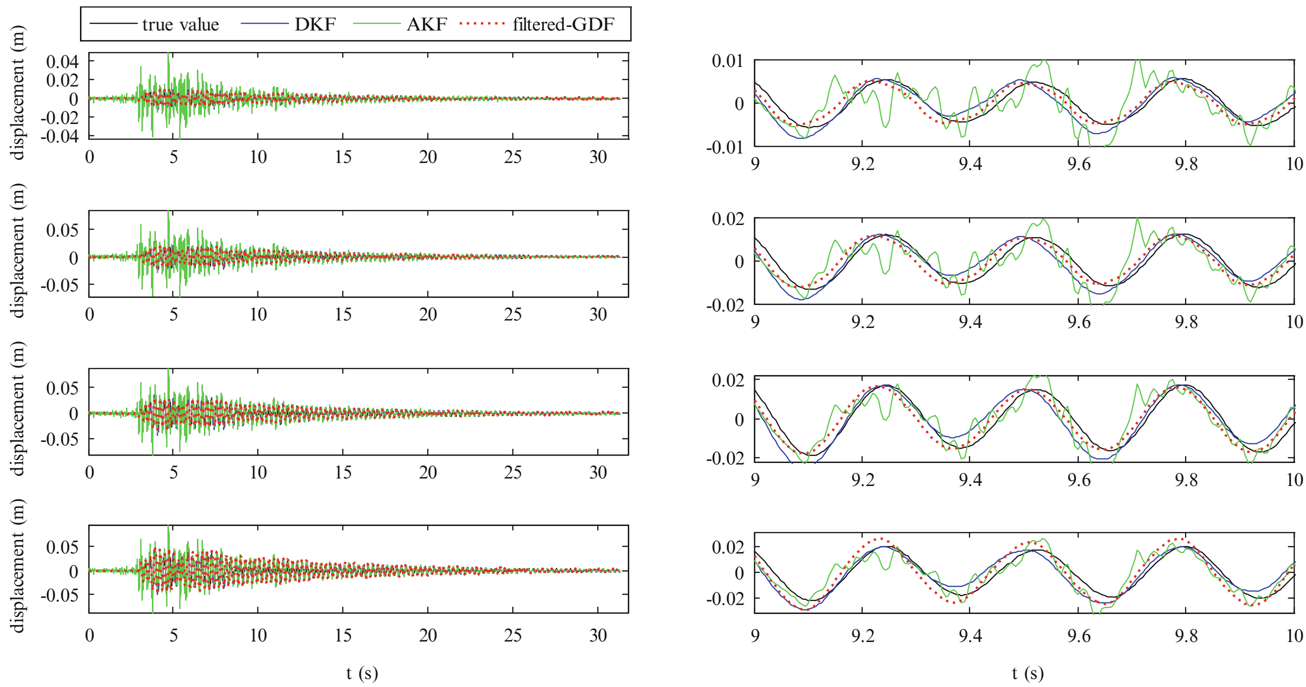


Fig. 1.10 Displacement time histories estimated by DKF, AKF and band pass filtered GDF when TSSID model is used for the floors 1, 2, .., 4 from top to bottom, respectively. The DKF and AKF are tuned for input estimation rather than the displacement, i.e. $\mathbf{Q}^{\mathbf{p}} = 10^{-3} \times \mathbf{I}$

the latter experimental campaign it is witnessed that when dealing with noisy acceleration measurements the GDF always features spurious low frequency components. Moreover, it is observed that limited DOF observations render the GDF prone to numerical instability and divergence. Concerning the AKF, it is shown that when fed with limited DOF acceleration measurements, the filter becomes sensitive to the tuning of the covariance of the fictitious noise process, in contrast to the proposed DKF algorithm.

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